

Thermodynamics of Gauss-Bonnet-Born-Infeld black holes in AdS space

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We construct solutions of a model which includes the Gauss-Bonnet and Born-Infeld terms for various horizon topologies ($k = 0, \pm 1$), and then the mass, temperature, entropy and heat capacity of black holes are computed. For the sake of simplicity, we perform the stability analysis of five dimensional topological black holes in AdS space.

I. INTRODUCTION

A renewed interest in the Lovelock gravity and Born-Infeld electrodynamics has appeared because they emerge in the low energy limit of string theory [1–3]. The effect of string theory on the left hand side of field equations of gravity is usually investigated by means of a low energy effective action which describes gravity at the classical level [4–7]. In addition to Einstein-Hilbert action, this effective action also involves higher derivative curvature terms. In the AdS/CFT correspondence, these higher derivative curvature terms correspond to the correction terms of large N expansion in the CFT side [8]. In general, the higher powers of curvature could give rise to a fourth or even higher order differential equation for the metric, and it would introduce ghosts and violate unitarity, therefore, the higher derivative terms may be a source of inconsistencies. However, the so-called Lovelock gravity is quite special

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[9]. Its Lagrangian is the sum of dimensionally extended Euler densities

$$\mathcal{L} = \sum_{n=0}^m \alpha_n \mathcal{L}_n, \quad (1)$$

where α_n , arbitrary constants, are the Lovelock coefficients and \mathcal{L}_n is the Euler density of a $2k$ -dimensional manifold

$$\mathcal{L}_n = \frac{1}{2^n} \delta_{c_1 b_1 \dots c_n d_n}^{a_1 b_1 \dots a_n b_n} R^{c_1 d_1}_{a_1 b_1} \dots R^{c_n d_n}_{a_n b_n}. \quad (2)$$

Here the generalized delta function $\delta_{c_1 \dots d_n}^{a_1 \dots b_n}$ is totally antisymmetric in both sets of indices and R^{cd}_{ab} is the Riemann tensor. Though the Lagrangian of Lovelock gravity consists of some higher derivative curvature terms, its field equations of motion contain the most symmetric conserved tensor with no more than two derivative of the metric. They have also been shown to be ghost-free when expanding about flat space, evading any problem with unitarity [10, 11]. In this paper, we indulge ourselves to with the first three terms of the Lovelock gravity, corresponding to the cosmological term, Einstein and Gauss-Bonnet terms, respectively. It has been argued that the Gauss-Bonnet term appears as the leading correction to the low energy effective action of the string theory and its Lagrangian is given by

$$\mathcal{L}_{GB} = R_{\gamma\delta\lambda\sigma} R^{\gamma\delta\lambda\sigma} - 4R_{\gamma\delta} R^{\gamma\delta} + R^2. \quad (3)$$

In Gauss-Bonnet gravity, static and spherically symmetric black hole solutions were firstly presented in [12], and of the charged in [13]. The thermodynamics of these solutions have been investigated in [14], of solutions with nontrivial topology in [15] and the charged black hole solutions in [16].

Besides the curvature terms, one would also expect higher derivative gauge field contributions to the action. This is done by explicitly constructing black holes solutions coupled to a Born-Infeld gauge field in the presence of a cosmological constant. The Born-Infeld electrodynamics is the nonlinear generalization of the Reissner-Nordström black hole (RNAdS) and is characterized by charged Q , mass M and the nonlinear parameter β . Its Lagrangian $\mathcal{L}(\mathcal{F})$ is given by

$$\mathcal{L}(\mathcal{F}) = 4\beta^2 \left(1 - \sqrt{1 + \frac{F^{\mu\nu} F_{\mu\nu}}{2\beta^2}} \right), \quad (4)$$

where the constant β is the Born-Infeld parameter, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is electromagnetic tensor field and A_μ is the vector potential. The Born-Infeld theory was originally introduced

to get a classical theory of charged particles with finite self-energy [17]. Hoffmann [18] was the first one in relating general relativity and the Born-Infeld electromagnetic field. He obtained a solution of the Einstein equations for a point-like Born-Infeld charge, while is devoid of the divergence of the metric at the origin that characterizes the Reissner-Nordström solution [19]. But, a conical singularity remained there, as it was later objected by Einstein and Rosen. The Born-Infeld black hole with zero cosmological constant was obtained by Garcia *et al* [20]. Later, The spherically symmetric Einstein-Born-Infeld black hole solutions with cosmological constant were studied in [21], Born-Infeld-dilaton models in [22]. Note that the work of the BIAdS in the grand canonical ensemble was carried out in [23]. The Euclidean action for the grand canonical ensemble was computed with the appropriate boundary terms. The thermodynamical quantities such as the Gibbs free energy, entropy and specific heat of the black holes are derived from it. For the Lovelock gravity, the asymptotically flat Born-Infeld black hole solutions in Gauss-Bonnet gravity were found in [24], five dimensional AdS black hole solutions in [19]. The Born-Infeld black hole solutions in third order Lovelock gravity have been obtained in [25]. The rotating Born-Infeld black hole solutions have been analyzed by Dehghani *et al* in general relativity [26], Gauss-Bonnet gravity [27] and third order Lovelock gravity [25], respectively. In this paper, we are concerned with the Born-Infeld-anti-de Sitter black hole (BIAdS) in Gauss-Bonnet gravity and discuss the thermodynamic quantities of five dimensional black holes in the canonical ensemble.

The structure of this paper is as follows. In section II, we present $n+1$ dimensional BIAdS black hole solutions in Gauss-Bonnet gravity. Then, the thermodynamics of these black holes will be discussed for a fixed-charge. In order to simplify the analysis of the thermodynamic properties, we study Gauss-Bonnet-Born-Infeld black holes in five dimensional spacetimes and investigate the stability of black holes by computing heat capacity of black holes in section III. Section IV devotes to concluding remarks.

II. GAUSS-BONNET-BORN-INFELD BLACK HOLES IN ADS SPACE

The action of Gauss-Bonnet gravity in the presence of nonlinear Born-Infeld electromagnetic field can be written as

$$\mathbf{I} = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} [-2\Lambda + R + \alpha \mathcal{L}_{GB} + \mathcal{L}(\mathcal{F})], \quad (5)$$

where $\Lambda = -\frac{n(n-1)}{2l^2}$ is a negative cosmological constant, α is the Gauss-Bonnet coefficient with dimension $(length)^2$ and is positive in the heterotic string theory. In the limit $\beta \rightarrow \infty$, $\mathcal{L}(\mathcal{F})$ Eq. (4) reduces to the standard Maxwell form

$$\mathcal{L}(\mathcal{F}) = -F^{\mu\nu}F_{\mu\nu} + \mathcal{O}(F^4). \quad (6)$$

By varying the action Eq. (5) with regard to the gauge field A_μ and the metric $g_{\mu\nu}$, these corresponding equations of motion are expressed as

$$\partial_\mu \left(\frac{\sqrt{-g}F^{\mu\nu}}{\sqrt{1 + \frac{F^2}{2\beta^2}}} \right) = 0 \quad (7)$$

and

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = & \frac{n(n-1)}{2l^2}g_{\mu\nu} + \alpha \left[\frac{1}{2}g_{\mu\nu}(R_{\gamma\delta\lambda\sigma}R^{\gamma\delta\lambda\sigma} - 4R_{\gamma\delta}R^{\gamma\delta} + R^2) \right. \\ & - 2RR_{\mu\nu} + 4R_{\mu\gamma}R^\gamma_\nu + 4R_{\gamma\delta}R^\gamma_\mu R^\delta_\nu - 2R_{\mu\gamma\delta\lambda}R^\gamma_{\nu}{}^{\delta\lambda} \left. \right] \\ & + \frac{2F_{\mu\lambda}F_\nu{}^\lambda}{\sqrt{1 + \frac{F_{\mu\lambda}F_\nu{}^\lambda}{2\beta^2}}} + \frac{1}{2}g_{\mu\nu}\mathcal{L}(\mathcal{F}). \end{aligned} \quad (8)$$

We assume the metric being of the following form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2h_{ij}dx^i dx^j, \quad (9)$$

where the coordinates are labelled as $x^\mu = (t, r, x^i)$, $(i = 1, \dots, (n-1))$. The metric function h_{ij} is a function of the coordinates x^i which span an $(n-1)$ -dimensional hypersurface with constant scalar curvature $(n-1)(n-2)k$. The constant k characterizes the geometric property of hypersurface and takes values $k = 0$ (flat), $k = -1$ (negative curvature) and $k = 1$ (positive curvature). $f(r)$ is an unknown function of r which will be determined later.

In the static and symmetric background Eq. (9), a class of solutions of Eq. (7) can be written down where all the components of $F^{\mu\nu}$ are zero except F^{rt} [21]

$$F^{rt} = \frac{\sqrt{(n-1)(n-2)}\beta q}{\sqrt{2\beta^2 r^{2n-2} + (n-1)(n-2)q^2}}, \quad (10)$$

where q is an integration constant relating to the electric charged of the solution. This can be verified from the behavior of F^{rt} in the limit $\beta \rightarrow \infty$ as $F^{rt} \sim \frac{q}{r^{n-1}}$. Via $Q = \frac{1}{4\pi} \int *F d\Omega$, the charge of the black hole can be found by computing the flux of the electric field at infinity

$$Q = \frac{q\Sigma_k}{4\pi} \sqrt{\frac{(n-1)(n-2)}{2}}, \quad (11)$$

where Σ_k represents the volume of constant curvature hypersurface described by $h_{ij}dx^i dx^j$.

Substituting the metric Eq. (9) and Eq. (10) into Eq. (8), one may use any components of Eq. (8) to find the function $f(r)$. The simplest equation is the rr -component of these equations which can be written as

$$\begin{aligned} \frac{n(n-1)}{l^2} - 4\beta^2(\sqrt{1+\eta} - 1) &= f(r)'[2\alpha(n-2)(n-3)(k-f(r))/r^3 \\ &+ 1/r](n-1) - [\alpha(n-3)(n-4)(k-f(r))/r^4 \\ &+ 1/r^2](n-1)(n-2)(k-f(r)), \end{aligned} \quad (12)$$

where prime denotes the derivative with regard to r and the function η is $\frac{(n-1)(n-2)}{2\beta^2 r^{2n-2}}q^2$. Then, the solution of Eq. (12) is given by

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}}(1 \pm \sqrt{g(r)}), \quad (13)$$

where the function $g(r)$ is demonstrated by using a hypergeometric function

$$\begin{aligned} g(r) &= 1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^n} - \frac{16\tilde{\alpha}\beta^2}{n(n-1)} \\ &+ \frac{8\sqrt{2}\tilde{\alpha}\beta}{n(n-1)r^{n-1}}\sqrt{2\beta^2 r^{2n-2} + (n-1)(n-2)q^2} \\ &- \frac{8(n-1)\tilde{\alpha}q^2}{nr^{2n-2}} \times {}_2F_1\left[\frac{n-2}{2n-2}, \frac{1}{2}, \frac{3n-4}{2n-2}, -\frac{(n-1)(n-2)q^2}{2\beta^2 r^{2n-2}}\right]. \end{aligned}$$

Here $\tilde{\alpha} = (n-2)(n-3)\alpha$ and m is an integration constant which is related to mass of the solution. In our convention, the ADM mass M is $\frac{(n-1)\Sigma_k m}{16\pi}$. Obviously, the Gauss-Bonnet-BIAdS black hole is characterized by charged Q , mass M , nonlinear parameter β , parameter l and the coefficient $\tilde{\alpha}$ of Gauss-Bonnet term. Note that the function $g(r)$ in Eq. (13) is negative for very small r . In order to maintain the non-negativity for $g(r)$, we set that r is larger than a minimum $r(r_{min})$, where r_{min} is the root of $g(r)$.

The hyper-surface $r = r_{min}$ is essentially a curvature singularity and the given solution will be a black hole solution if this singular hyper-surface is surrounded by the event horizon (having radius r_+ such that $f(r_+) = 0$), otherwise, the solution describes a naked singularity. In addition, here we only focus on the positive coefficient $\tilde{\alpha}$. When $k = 1$ and $1/l^2 = 0$, the solution is asymptotically flat solution if one take "-" sign. But it is asymptotically anti-de Sitter solution with a negative gravitational mass for the "+" sign, indicating the instability. Hence, we will only consider the branch with "-" branch. It can be checked that for $n = 3$, it reduces to the solution of [19].

Based on Eq. (10), the electric field F_{rt} is obtained

$$F_{rt} = -\frac{\sqrt{(n-1)(n-2)}\beta q}{\sqrt{2\beta^2 r^{2n-2} + (n-1)(n-2)q^2}}. \quad (14)$$

Then, according to $F_{rt} = \partial_r A_t - \partial_t A_r$, the electric gauge potential is given by

$$A_t = \sqrt{\frac{n-1}{2n-4}} \frac{q}{r^{n-2}} F(\eta) - \Phi, \quad (15)$$

where Φ is a constant and function $F(\eta)$ is $F(\eta) = {}_2F_1\left[\frac{n-2}{2n-2}, \frac{1}{2}, \frac{3n-4}{2n-2}, -\frac{(n-1)(n-2)q^2}{2\beta^2 r^{2n-2}}\right]$. In order to fix the gauge potential at the horizon to be zero, the quantity Φ is chosen as

$$\Phi = \sqrt{\frac{n-1}{2n-4}} \frac{q}{r_+^{n-2}} F(\eta_+). \quad (16)$$

Clearly, the quantities A_t and Φ in Gauss-Bonnet gravity is identical with corresponding ones [21] in general relativity.

In the limit $\tilde{\alpha} \rightarrow 0$, the black hole solution Eq. (13) tends to

$$\begin{aligned} f(r) = & k + \frac{r^2}{l^2} - \frac{m}{r^{n-2}} + \frac{4\beta^2 r^2}{n(n-1)} - \frac{2\sqrt{2}\beta}{n(n-1)r^{n-3}} \\ & \times \sqrt{2\beta^2 r^{2n-2} + (n-1)(n-2)q^2} + \frac{2(n-1)q^2}{nr^{2n-4}} F(\eta). \end{aligned} \quad (17)$$

one can find that it corresponds to the Einstein-Born-Infeld black hole solution in AdS space [21]. While, in the limit $q \rightarrow 0$, the solution Eq. (13) reduces to uncharged Gauss-Bonnet solution

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}} \left(1 - \sqrt{1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^n}}\right). \quad (18)$$

On the other hand, if taking $\beta \rightarrow \infty$ in Eq. (13), the solution $f(r)$ reduces to the Lovelock-Maxwell solution found in [13]

$$f(r) = 1 + \frac{r^2}{2\tilde{\alpha}} \left(1 - \sqrt{1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^n} - \frac{4\tilde{\alpha}q^2}{r^{2n-2}}}\right).$$

In the rest of the section, we discuss the thermodynamic properties of black holes. In terms of horizon radius r_+ , the ADM mass M is obtained

$$\begin{aligned} M = & \frac{(n-1)\Sigma_k r_+^{n-2}}{16\pi} \left\{ k + \frac{\tilde{\alpha}k^2}{r_+^2} + \frac{r_+^2}{l^2} + \frac{4\beta^2 r_+^2}{n(n-1)} - \frac{2\sqrt{2}\beta}{n(n-1)r_+^{n-3}} \right. \\ & \times \left. \sqrt{2\beta^2 r_+^{2n-2} + (n-1)(n-2)q^2} + \frac{(n-1)}{n} \frac{2q^2}{r_+^{2n-4}} F(\eta_+) \right\}. \end{aligned} \quad (19)$$

The Hawking temperature of black holes can be obtained by requirement of the absence of conical singularity at the event horizon in the Euclidean section of the black hole solution.

It can be written as $T_H = \frac{f'(r_+)}{4\pi}$

$$T_H = \frac{1}{4\pi r_+(r_+^2 + 2k\tilde{\alpha})} \left\{ \frac{nr_+^4}{l^2} + (n-2)kr_+^2 + (n-4)k^2\tilde{\alpha} + \frac{4\beta^2 r_+^4}{n-1} \right. \\ \left. - \frac{2\sqrt{2}\beta}{(n-1)r_+^{n-5}} \times \sqrt{2\beta^2 r_+^{2n-2} + (n-1)(n-2)q^2} \right\}. \quad (20)$$

We note that in the limit $\beta \rightarrow \infty$ and $q \neq 0$, T_H reduces to the charged case

$$\tilde{T}_H = \frac{nr_+^4 + (n-2)kl^2r_+^2 + (n-4)k^2\tilde{\alpha}l^2 - (n-2)q^2l^2/r_+^{2n-5}}{4\pi l^2 r_+(r_+^2 + 2k\tilde{\alpha})},$$

while in the limit of $q \rightarrow 0$, T_H reduces to uncharged case.

Another important thermodynamic quantity is the entropy of black hole. In general relativity, the entropy of black hole satisfy the so-called area formula, namely entropy equals to one quarter of horizon area [28]. However, the area law of entropy is not satisfied in general in higher derivative gravity [29]. Using the standard formula for entropy $S = \int T_H^{-1} (\frac{\partial M}{\partial r_+})_Q dr_+$, we get

$$S = \frac{\Sigma_k}{4} r_+^{n-1} \left[1 + \frac{(n-1)}{(n-3)} \frac{2\tilde{\alpha}k}{r_+^2} \right], \quad (21)$$

where

$$\left(\frac{\partial M}{\partial r_+} \right)_Q = \frac{(n-1)\Sigma_k}{16\pi} r_+^{n-5} \left[\frac{nr_+^4}{l^2} + (n-2)kr_+^2 + (n-4)k^2\tilde{\alpha} + \frac{4\beta^2 r_+^4}{n-1} \right. \\ \left. - \frac{2\sqrt{2}\beta}{(n-1)r_+^{n-5}} \sqrt{2\beta^2 r_+^{2n-2} + (n-1)(n-2)q^2} \right] \\ = \frac{(n-1)\Sigma_k}{4} r_+^{n-4} (r_+^2 + 2k\tilde{\alpha}) T_H. \quad (22)$$

In Eq. (19) r_+ is the real root of Eq. (21) which is a function of S . One may regard the parameter S and Q as a complete set extensive parameters for the mass $M(S, Q)$ and define the intensive parameters conjugate to them [25]. These quantities are temperature and the electric potential

$$T_H = \left(\frac{\partial M}{\partial S} \right)_Q, \quad \Phi = \left(\frac{\partial M}{\partial Q} \right)_S. \quad (23)$$

One can find that the intensive quantities T_H and Φ in Eq. (23) is consistent with Eq. (16) and Eq. (20), respectively. Hence, the relevant thermodynamic quantities Eq. (16) and

Eq. (20) satisfy the first law of thermodynamics

$$dM = T_H dS + \Phi dQ \quad (24)$$

The local stability of black hole is determined by the sign of its heat capacity. If the heat capacity is positive, the black hole is locally stable to thermal fluctuations. Otherwise the black hole is locally unstable. The heat capacity for a fixed-charge is expressed as

$$C_Q = \left(\frac{\partial M}{\partial T_H}\right)_Q = \left(\frac{\partial M}{\partial r_+}\right)_Q / \left(\frac{\partial T_H}{\partial r_+}\right)_Q, \quad (25)$$

where

$$\begin{aligned} \left(\frac{\partial T_H}{\partial r_+}\right)_Q = & \frac{1}{4\pi r_+^2 (r_+^2 + 2k\tilde{\alpha})^2} \left[\frac{nr_+^6}{l^2} + \frac{6n\tilde{\alpha}kr_+^4}{l^2} - (n-2)kr_+^4 - (n-8)k^2\tilde{\alpha}r_+^2 \right. \\ & - 2(n-4)k^3\tilde{\alpha}^2 \left. \right] + \frac{1}{(n-1)\pi(r_+^2 + 2k\tilde{\alpha})^2} \left\{ r_+^2(r_+^2 + 6k\tilde{\alpha})\beta^2 \right. \\ & \left. + \frac{\sqrt{2}\beta J(r)}{2r_+^{n-1}\sqrt{2\beta^2 r_+^{2n-2} + (n-1)(n-2)q^2}} \right\}, \end{aligned} \quad (26)$$

where $J(r) = (n-1)(n-2)q^2 r_+^2 ((n-2)r_+^2 + 2k(n-4)\tilde{\alpha}) - 2r_+^{2n}(r_+^2 + 6k\tilde{\alpha})\beta^2$.

III. STABILITY OF FIVE DIMENSIONAL BLACK HOLES

Note that the expression for heat capacity C_Q looks like very complicated. In order to perform the stability further, in this section, we explore the Gauss-Bonnet-Born-Infeld black holes in five dimension spacetimes. Then, the black hole solution Eq. (13) becomes

$$\begin{aligned} f(r) = & k + \frac{r^2}{2\tilde{\alpha}} \left\{ 1 - \left[1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^4} - \frac{4\tilde{\alpha}\beta^2}{3} \left(1 - \sqrt{1 + \frac{3q^2}{\beta^2 r^6}} \right) \right. \right. \\ & \left. \left. - \frac{6\tilde{\alpha}q^2}{r^6} \times {}_2F_1\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{3q^2}{\beta^2 r^6}\right]^{1/2} \right] \right\}. \end{aligned} \quad (27)$$

Using the Eq. (22) and Eq. (26), therefore, the heat capacity C_Q is obtained

$$\begin{aligned} C_Q = & \frac{3\Sigma_k r_+^3 (r_+^2 + 2k\tilde{\alpha})^2 \sqrt{1 + \frac{3q^2}{\beta^2 r_+^6}}}{\Upsilon + \Xi} \\ & \times [6r_+^2 + l^2(3k + 2\beta^2 r_+^2(1 - \sqrt{1 + \frac{3q^2}{\beta^2 r_+^6}}))], \end{aligned} \quad (28)$$

where $\Upsilon = 8r_+^4[(3+l^2\beta^2)\sqrt{1+\frac{3q^2}{\beta^2r_+^6}}-l^2\beta^2](r_+^2+6k\tilde{\alpha})$ and $\Xi = 12l^2[kr_+^2(2k\tilde{\alpha}-r_+^2)\sqrt{1+\frac{3q^2}{\beta^2r_+^6}}+4q^2]$. One can see that these thermodynamic properties drastically depend on the event horizon structure k . According to the classification of event horizon structures $k = 0$ and $k \pm 1$, below each case will be discussed respectively.

A. The case of $k = 0$

For $k = 0$ and $n = 4$, we have

$$T_H = \frac{1}{3\pi l^2 r_+^2}[(3+l^2\beta^2)r_+^3 - \beta l^2 \sqrt{\beta^2 r_+^6 + 3q^2}], \quad S = \frac{\Sigma_k r_+^3}{4}, \quad (29)$$

$$\begin{aligned} C_Q &= \frac{3\Sigma_k r_+^9[3+l^2\beta^2(1-\sqrt{1+\frac{3q^2}{\beta^2r_+^6}})]\sqrt{1+\frac{3q^2}{\beta^2r_+^6}}}{4r_+^6[(3+l^2\beta^2)\sqrt{1+\frac{3q^2}{\beta^2r_+^6}}-l^2\beta^2]+24l^2q^2} \\ &= \frac{3\Sigma_k r_+^6\sqrt{1+\frac{3q^2}{\beta^2r_+^6}}}{4r_+^6[(3+l^2\beta^2)\sqrt{1+\frac{3q^2}{\beta^2r_+^6}}-l^2\beta^2]+24l^2q^2} T_H. \end{aligned} \quad (30)$$

It is interesting to notice that these thermodynamic properties are independent of the coefficient $\tilde{\alpha}$ and the entropy of black hole is proportional to the area of the horizon. From the Eq. (29), the radius of the extremal black hole $T_H = 0$ is obtained $r_0 = \sqrt[6]{\frac{q^2\beta^2l^4}{3+2l^2\beta^2}}$ which is the only root of Eq. (29). In Fig. 1, we draw the temperatures T_H of black holes with different values of the parameter $\beta = 2, 1$ and 0.1 . For instance, the temperature T_H vanishes at $r_+ = r_0 \approx 0.76$ for $l = \beta = 1$. Then it goes to positive infinity as $r_+ \rightarrow \infty$. Furthermore, this similar trait of the temperatures T_H of black holes is universal for different values of parameter l . On the other hand, the heat capacity C_Q disappears for $T_H = 0$ which is presented in Eq. (30). One can see that C_Q is positive provided $r_+ > r_0$ (see Fig. 1). We therefore conclude that the black holes are stable against fluctuations as they are shown in [27].

Considering black holes in flat space, we obtain the temperature T_H of black holes in case of $l \rightarrow \infty$

$$T_H = \frac{\beta^2 r_+}{3\pi} \left(1 - \sqrt{1 + \frac{3q^2}{\beta^2 r_+^6}}\right), \quad (31)$$

Apparently the temperature T_H is always negative in the whole range of r_+ . Henceforth there don't exist black holes in flat space for $k = 0$.

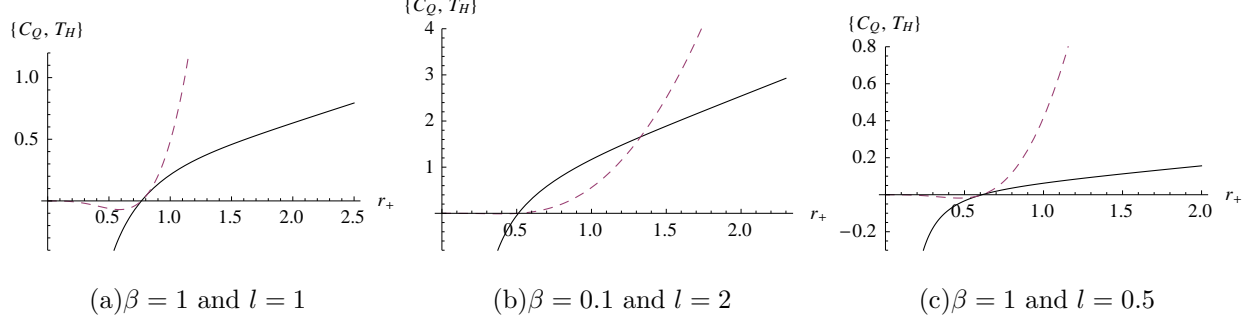


FIG. 1: Temperature T_H (solid curves) and heat capacity C_Q (dashed curves) versus horizon radius r_+ for $n = 4$, $q = 1$ and $k = 0$. Here we choose $\Sigma_k = 1$.

B. The case of $k = 1$

In this subsection we explore some physical aspects of the black holes with positive constant curvature hypersurface horizon. For $n = 4$, we find that this class of solutions was also studied in [19, 24]. While, we are concerned with the thermodynamic properties of black holes which have been never discussed before. One can see that there maybe exist extremal black holes for $T_H = 0$ in Eq. (20), which is expressed as

$$T_H = \frac{1}{6\pi l^2(r_{ext}^2 + 2\tilde{\alpha})} [6r_{ext}^3 + 3l^2 r_{ext} + 2l^2 \beta^2 r_{ext}^3 (1 - \sqrt{1 + \frac{3q^2}{\beta^2 r_{ext}^6}})] = 0. \quad (32)$$

For simplicity, here we set the parameters $l = 1$ and $q = 1$. Then, the Eq. (32) can be rewritten as

$$4(3 + 2\beta^2)R_{ext}^3 + 4(3 + \beta^2)R_{ext}^2 + 3R_{ext} - 4\beta^2 = 0. \quad (33)$$

where $r_{ext}^2 = R_{ext}$, namely $R_{ext} \geq 0$. Then, the discriminant of this cubic equation is given by

$$\Delta = 48\beta^2(216 + 4095\beta^2 + 5040\beta^4 + 1664\beta^6). \quad (34)$$

Clearly, it is always positive for parameter β . Therefore, there only exist one positive real root of this cubic equation. The radius of the event horizon for extremal black hole is given by

$$r_{ext}^2 = \frac{1}{6(3 + 2\beta^2)} \left\{ -2(3 + \beta^2) + [J(r) - 3\beta(3 + 2\beta^2)\sqrt{\Gamma(\beta)}]^{1/3} + [J(\beta) + 3\beta(3 + 2\beta^2)\sqrt{\Gamma(\beta)}]^{1/3} \right\}, \quad (35)$$

where $\Gamma(\beta) = 3(216 + 4095\beta^2 + 5040\beta^4 + 1664\beta^6)$ and $J(\beta) = 27 + 999\beta^2 + 1278\beta^4 + 424\beta^6$. In Fig. 2, we plot the temperature T_H with different values of parameters $\tilde{\alpha}$ and β . Obviously, the temperature T_H vanishes at $r_+ = r_{ext}$, and then goes to positive infinity as radius increases. This trait does not change even though the black hole solutions take various values for parameters $\tilde{\alpha}$ and β .

Based on Eq. (19), therefore the mass of extremal black hole is expressed in terms of r_{ext} as

$$M_{ext} = \frac{3\Sigma_k r_{ext}^2}{16\pi} \left\{ 1 + \frac{\tilde{\alpha}}{r_{ext}^2} + r_{ext}^2 + \frac{\beta^2 r_{ext}^2}{3} - \frac{\beta}{3r_{ext}} \sqrt{\beta^2 r_{ext}^6 + 3} \right. \\ \left. + \frac{3}{2r_{ext}^4} \times {}_2F_1\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{3}{\beta^2 r_{ext}^6}\right] \right\}. \quad (36)$$

Note that if $m > m_{ext}$, there are more than one horizon while there will be degenerate horizon at $r = r_{ext}$ for $m = m_{ext}$. But for $m < m_{ext}$, no horizon exists and we are left with a naked singularity.

With regard to the stability of black holes, C_Q has been demonstrated by $(\frac{\partial M}{\partial r_+})_Q / (\frac{\partial T_H}{\partial r_+})_Q$ in Eq. (25). Since the function $(\frac{\partial M}{\partial r_+})_Q$ is always non-negative for $T_H \geq 0$, the sign of heat capacity C_Q is determined by the one of function $(\frac{\partial T_H}{\partial r_+})_Q$. From Fig. 2, one see that T_H vanishes at $r_+ = r_{ext}$, and then goes to positive infinity as radius r_+ increases. Therefore, C_Q is always positive when $r_+ > r_{ext}$. In this case, the black holes are locally stable in the region $r_+ > r_{ext}$.

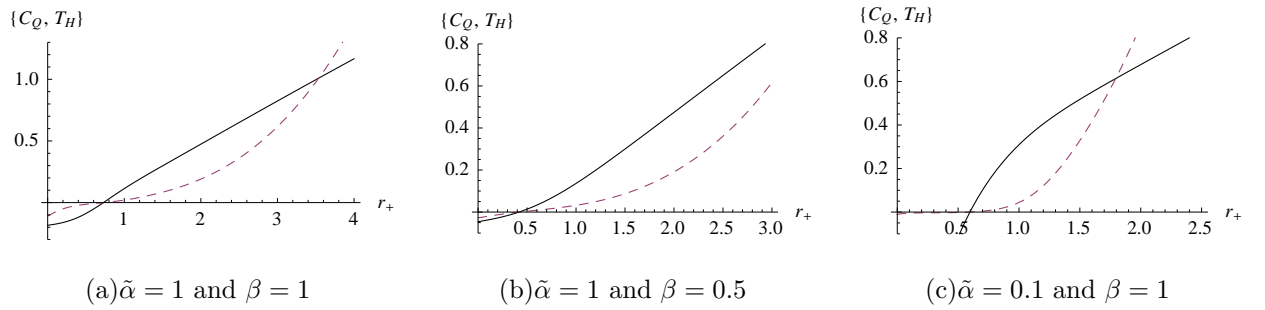


FIG. 2: Temperature T_H (solid curves) and heat capacity C_Q (dashed curves) versus horizon radius r_+ for $l = 1$, $n = 4$, $q = 1$ and $k = 1$. Here we choose $\Sigma_k = 0.01$.

In case of $\Lambda \rightarrow 0$, namely $l^2 \rightarrow \infty$, the Gauss-Bonnet-Born-Infeld black holes reduces to the flat case. And then, a asymptotically flat black hole solution can be easily obtained by

taking $k = 1$. Hence, the temperature \bar{T}_H of this new solution is given by

$$\bar{T}_H = \frac{r_+}{6\pi(r_+^2 + 2\tilde{\alpha})} [3 + 2\beta^2 r_+^2 (1 - \sqrt{1 + \frac{3q^2}{\beta^2 r_+^6}})]. \quad (37)$$

The extremal radius \bar{r}_{ext} can be straightforwardly written as $\bar{r}_{ext} = \frac{(\sqrt{9+64\beta^4}-3)^{1/2}}{2\sqrt{2}\beta}$. According to Eq. (19), the mass of black hole in case of $l \rightarrow \infty$ is given by

$$\begin{aligned} \bar{M}_{ext} = & \frac{3\Sigma_k \bar{r}_{ext}^2}{16\pi} \left\{ 1 + \frac{\tilde{\alpha}}{\bar{r}_{ext}^2} + \frac{\beta^2 \bar{r}_{ext}^2}{3} - \frac{\beta}{3\bar{r}_{ext}} \sqrt{\beta^2 \bar{r}_{ext}^6 + 3} \right. \\ & \left. + \frac{3}{2\bar{r}_{ext}^4} \times {}_2F_1\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{3}{\beta^2 \bar{r}_{ext}^6}\right] \right\}. \end{aligned} \quad (38)$$

Different from the AdS case, the temperature \bar{T}_H starts from zero at $r_+ = \bar{r}_{ext}$ increases sharply reaches a local maximum at $r_+ = r_m$, and then decreases gradually to zero as $r_+ \rightarrow \infty$. The temperature \bar{T}_H with different values of parameter β and coefficient $\tilde{\alpha}$ is plotted in Fig. 3. Based on Eq. (25) $C_Q = (\frac{\partial M}{\partial r_+})_Q / (\frac{\partial T_H}{\partial r_+})_Q$, the sign of heat capacity C_Q is only determined by $(\frac{\partial \bar{T}_H}{\partial r_+})_Q$ and C_Q also disappears when \bar{T}_H is equal to zero. Therefore, the C_Q maintains positive in the region $r_{ext} < r_+ < r_m$, and then becomes negative in the region $r_+ > r_m$. As a result, the black holes are locally stable in the domain $r_{ext} < r_+ < r_m$ and locally unstable provided $r_+ > r_m$. Obviously, the cosmological constant Λ plays an important role in the distribution of stable regions of Gauss-Bonnet-Born-Infeld black holes.

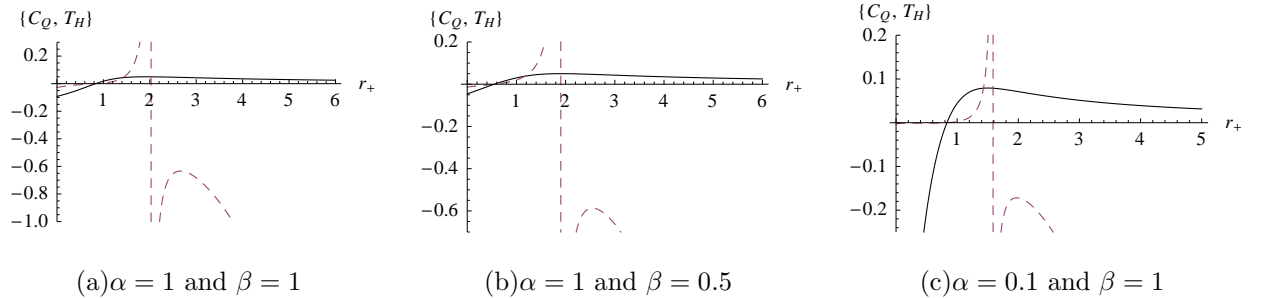


FIG. 3: Temperature T_H (solid curves) and heat capacity C_Q (dashed curves) versus horizon radius r_+ for $n = 4$, $q = 1$ and $k = 1$. Here we choose $\Sigma_k = 1/800$.

C. The case of $k = -1$

Now, we turn to the case the horizon is a negative constant curvature hypersurface. In five dimensional spacetimes, the temperature of black holes Eq. (20) becomes

$$T_H = \frac{1}{6\pi l^2(r_+^2 - 2\tilde{\alpha})}[6r_+^3 - 3l^2r_+ + 2l^2\beta^2r_+^3(1 - \sqrt{1 + \frac{3q^2}{\beta^2r_+^6}})]. \quad (39)$$

Then, the existence of extremal black holes depends on the existence of positive root(s) for $T_H = 0$, which reduces to

$$6r_{ext}^3 - 3l^2r_{ext} + 2l^2\beta^2r_{ext}^3 - 2l^2\beta\sqrt{\beta^2r_{ext}^6 + 3q^2} = 0. \quad (40)$$

Adopted the same approach above, we order the parameters $l = 1$ and $q = 1$. Then, the Eq. (40) can be rewritten as

$$4(3 + 2\beta^2)R_{ext}^3 - 4(3 + \beta^2)R_{ext}^2 + 3R_{ext} - 4\beta^2 = 0, \quad (41)$$

where R_{ext} is defined as r_{ext}^2 , namely, $R_{ext} \geq 0$. Then, the discriminant of this cubic equation is

$$\Delta_* = 48\beta^2(-216 + 3663\beta^2 + 5328\beta^4 + 1792\beta^6). \quad (42)$$

Different from the case $k = 1$, the discriminant Δ_* can be take negative or positive values for different values of parameter β . However, the root of Eq. (42) is not real and is located between 0.2335 and 0.2336. Therefore, we discuss the two case respectively.

If the parameter $\beta \geq 0.2336$, the discriminant Δ_* is positive. Then, the only one real root is obtained

$$r_{ext}^2 = \frac{1}{12(3 + 2\beta^2)} \left\{ 4(3 + \beta^2) + [J(\beta) - 6(3 + 2\beta^2)\sqrt{\Delta_*}]^{1/3} + [J(\beta) + 6(3 + 2\beta^2)\sqrt{\Delta_*}]^{1/3} \right\}, \quad (43)$$

where $J(\beta) = 8(-27 + 945\beta^2 + 1314\beta^4 + 440\beta^6)$. In case of $\beta \leq 0.2335$, namely, $\Delta_* < 0$, there exist three different real roots. They are give by

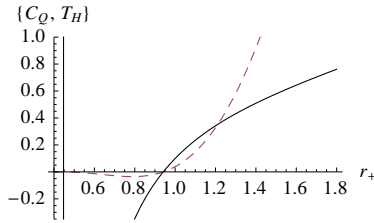
$$\begin{aligned} r_{e1}^2 &= \frac{1}{3(3 + 2\beta^2)}[3 + \beta^2 - \sqrt{9 + 6\beta^2 + 4\beta^4} \cos(\theta/3)], \\ r_{e2}^2 &= \frac{1}{3(3 + 2\beta^2)} \left\{ 3 + \beta^2 + \frac{1}{2}\sqrt{9 + 6\beta^2 + 4\beta^4}[\cos(\theta/3) - \sqrt{3}\sin(\theta/3)] \right\}, \\ r_{e3}^2 &= \frac{1}{3(3 + 2\beta^2)} \left\{ 3 + \beta^2 + \frac{1}{2}\sqrt{9 + 6\beta^2 + 4\beta^4}[\cos(\theta/3) + \sqrt{3}\sin(\theta/3)] \right\}, \end{aligned} \quad (44)$$

where $\theta = \arccos U$, $U = -\frac{-27+945\beta^2+1314\beta^4+440\beta^6}{(9+6\beta^2+4\beta^4)^{3/2}}$. It is worth to mention that the quantities r_{e1}^2 , r_{e2}^2 and r_{e3}^2 is only the three roots of Eq. (41). But not all roots satisfy the equation of temperature T_H Eq. (39). We find that only r_{e3}^2 is the root of Eq. (39). In addition, the horizon radius must obey

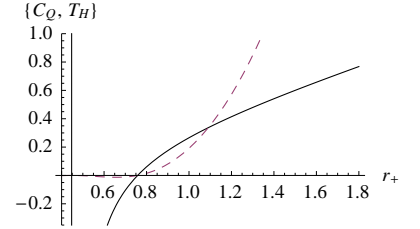
$$r_+^2 \geq 2\tilde{\alpha}. \quad (45)$$

Therefore, the radius of extremal black hole has a constraint $r_{ext}^2 \geq 2\tilde{\alpha}$. The graph of T_H is plotted in Fig. 4. The temperature T_H blows up at $r_+ = \sqrt{2\tilde{\alpha}}$ and changes sign at $r_+ = r_{ext}$ ($\Delta_* > 0$) or $r_+ = r_{e3}$ ($\Delta_* < 0$). Then, it gradually increases as $r_+ \rightarrow \infty$.

Now, let us discuss the stability of black holes. In Fig. 4, we have plotted C_Q as a function of horizon radius r_+ . We see from the figure that the heat capacity C_Q is positive if r_+ is larger than the radius of extremal black hole r_{ext} . It means that the black holes are locally stable in the region $r_+ > r_{ext}$.



(a) $\tilde{\alpha} = 0.1$ and $\Delta_* > 0$ ($\beta = 1$)



(b) $\tilde{\alpha} = 0.1$ and $\Delta_* < 0$ ($\beta = 0.1$)

FIG. 4: Temperature T_H (solid curves) and heat capacity C_Q (dashed curves) versus horizon radius r_+ for $n = 4$, $q = 1$, $l = 1$ and $k = -1$. Here we choose $\Sigma_k = 1$.

However, the case of $\Lambda = 0$ is quite different from corresponding one above. Here we also let $q = 1$. Then, the temperature T_H is

$$T_H = \frac{1}{6\pi(r_+^2 - 2\tilde{\alpha})}[-3r_+ + 2\beta^2 r_+^3(1 - \sqrt{1 + \frac{3}{\beta^2 r_+^6}})]. \quad (46)$$

Since the horizon radius also satisfies $r_+^2 \geq 2\tilde{\alpha}$, we find that this equation Eq. (46) does not have positive root. Hence, we conclude that for $k = -1$, the asymptotically flat black holes are unstable in the whole range of r_+ .

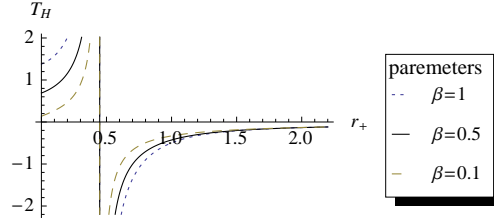


FIG. 5: Temperature T_H versus r_+ for $n = 4$, $q = 1$, $\alpha = 0.1$ and $k = -1$.

IV. CONCLUDING REMARKS

In this paper, we have constructed static topological black hole solutions of Gauss-Bonnet-Born-Infeld action in the presence of cosmological constant. Then, we discussed the thermodynamic properties of black holes including gravitational mass, Hawking temperature and entropy of black holes. We also notice that the entropy of Gauss-Bonnet-Born-Infeld black holes get no correction from the Born-Infeld gauge field.

Later, we performed the stability analysis of these topological black holes in five dimensional spacetimes for convenience. For $k = 0$, all the thermodynamic quantities of the black holes don't depend on the Gauss-Bonnet coefficient $\tilde{\alpha}$ and are the same as those of black holes in Einstein-Born-Infeld gravity although the two black hole solutions are quite different. For the horizon is negative constant hypersurface, the cosmological constant Λ plays an important role in the distribution of regions for the stability of black holes. the Gauss-Bonnet-Born-Infeld black holes in AdS space are thermodynamically stable provided $r_+ > r_{ext}$, while corresponding ones in flat space are unstable in the whole range of r_+ . For the horizon is positive constant hypersurface, unlike the uncharged Gauss-Bonnet black holes that there don't exist extremal black holes when $k = 1$ [15], the extremal black holes also exist in Gauss-Bonnet-Born-Infeld gravity. In addition, the asymptotically flat Gauss-Bonnet-Born-Infeld black holes are thermodynamically stable in the region $r_+ > r_{ext}$. However, since the existence of cosmological constant Λ , the black holes become unstable beyond the location $r_+ = r_m$ where the temperature reaches local maximum.

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